Maple 2018.2 Integration Test Results
on the problems in "6 Hyperbolic functions/6.5 Hyperbolic secant"

Test results for the 6 problems in "6.5.1 (c+d x)^m (a+b sech)^n.txt"

Problem 1: Unable to integrate problem.

$$\int (d x + c)^3 \operatorname{sech}(b x + a)\, dx$$

Optimal(type 4, 160 leaves, 9 steps):

$$\frac{2 (d x + c)^3 \arctan(e^{b x + a})}{b} - \frac{3 I d (d x + c)^2 \operatorname{polylog}(2, -I e^{b x + a})}{b^2} + \frac{3 I d (d x + c)^2 \operatorname{polylog}(2, I e^{b x + a})}{b^2} + \frac{6 I d^2 (d x + c) \operatorname{polylog}(3, -I e^{b x + a})}{b^3} - \frac{6 I d^2 (d x + c) \operatorname{polylog}(3, I e^{b x + a})}{b^3} - \frac{6 I d^3 \operatorname{polylog}(4, -I e^{b x + a})}{b^4} + \frac{6 I d^3 \operatorname{polylog}(4, I e^{b x + a})}{b^4}$$

Result(type 8, 16 leaves):

$$\int (d x + c)^3 \operatorname{sech}(b x + a)\, dx$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int (d x + c) \operatorname{sech}(b x + a)\, dx$$

Optimal(type 4, 54 leaves, 5 steps):

$$\frac{2 (d x + c) \arctan(e^{b x + a})}{b} - \frac{I d \operatorname{polylog}(2, -I e^{b x + a})}{b^2} + \frac{I d \operatorname{polylog}(2, I e^{b x + a})}{b^2}$$

Result(type 4, 448 leaves):

$$\frac{I d \operatorname{dilog}(-I \cosh(b x + a) - I \sinh(b x + a))}{b^2} - \frac{I d \ln\left((1 - I) \cosh\left(\frac{b x}{2} + \frac{a}{2}\right) + (1 + I) \sinh\left(\frac{b x}{2} + \frac{a}{2}\right)\right) x}{b} + \frac{I d \ln(-I \cosh(b x + a) - I \sinh(b x + a)) \ln\left((1 - I) \cosh\left(\frac{b x}{2} + \frac{a}{2}\right) + (1 + I) \sinh\left(\frac{b x}{2} + \frac{a}{2}\right)\right)}{b^2} - \frac{I d \ln\left((1 - I) \cosh\left(\frac{b x}{2} + \frac{a}{2}\right) + (1 + I) \sinh\left(\frac{b x}{2} + \frac{a}{2}\right)\right) a}{b^2} - \frac{I d \operatorname{dilog}(I \cosh(b x + a) + I \sinh(b x + a))}{b^2} + \frac{I d \ln\left((1 + I) \cosh\left(\frac{b x}{2} + \frac{a}{2}\right) + (1 - I) \sinh\left(\frac{b x}{2} + \frac{a}{2}\right)\right) x}{b} - \frac{I d \ln(I \cosh(b x + a) + I \sinh(b x + a)) \ln\left((1 + I) \cosh\left(\frac{b x}{2} + \frac{a}{2}\right) + (1 - I) \sinh\left(\frac{b x}{2} + \frac{a}{2}\right)\right)}{b^2}$$

$$+\frac{I\,d\ln\left((1+I)\cosh\left(\frac{b x}{2}+\frac{a}{2}\right)+(1-I)\sinh\left(\frac{b x}{2}+\frac{a}{2}\right)\right)a}{b^2}+\frac{I\,d\ln(-I\cosh(b x+a)-I\sinh(b x+a))\,x}{2\,b}$$
$$-\frac{I\,d\ln(I\cosh(b x+a)+I\sinh(b x+a))\,x}{2\,b}+\frac{I\,d\ln(-I\cosh(b x+a)-I\sinh(b x+a))\,a}{2\,b^2}-\frac{I\,d\ln(I\cosh(b x+a)+I\sinh(b x+a))\,a}{2\,b^2}$$
$$-\frac{2\,d\,a\arctan\left(e^{b x+a}\right)}{b^2}+\frac{2\,c\arctan\left(e^{b x+a}\right)}{b}$$

```
Problem 4: Unable to integrate problem.
```

$$\int (d x+c)^2\operatorname{sech}(b x+a)^3\,dx$$

```
Optimal(type 4, 160 leaves, 9 steps):
```

$$\frac{(d x+c)^2\arctan\left(e^{b x+a}\right)}{b}-\frac{d^2\arctan(\sinh(b x+a))}{b^3}-\frac{I\,d\,(d x+c)\operatorname{polylog}\left(2,-I\,e^{b x+a}\right)}{b^2}+\frac{I\,d\,(d x+c)\operatorname{polylog}\left(2,I\,e^{b x+a}\right)}{b^2}$$
$$+\frac{I\,d^2\operatorname{polylog}\left(3,-I\,e^{b x+a}\right)}{b^3}-\frac{I\,d^2\operatorname{polylog}\left(3,I\,e^{b x+a}\right)}{b^3}+\frac{d\,(d x+c)\operatorname{sech}(b x+a)}{b^2}+\frac{(d x+c)^2\operatorname{sech}(b x+a)\tanh(b x+a)}{2\,b}$$

```
Result(type 8, 182 leaves):
```

$$\frac{e^{b x+a}\left(b d^2 x^2\left(e^{b x+a}\right)^2+2 b c d x\left(e^{b x+a}\right)^2+b c^2\left(e^{b x+a}\right)^2-b d^2 x^2+2 d^2 x\left(e^{b x+a}\right)^2-2 b c d x+2 c d\left(e^{b x+a}\right)^2-b c^2+2 d^2 x+2 c d\right)}{b^2\left(\left(e^{b x+a}\right)^2+1\right)^2}+8\left(\int\frac{e^{b x+a}\left(b^2 d^2 x^2+2 b^2 c d x+b^2 c^2-2 d^2\right)}{8\,b^2\left(\left(e^{b x+a}\right)^2+1\right)}\,dx\right)$$

```
Problem 5: Unable to integrate problem.
```

$$\int\left(\frac{x}{\operatorname{sech}(x)^{7/2}}-\frac{5 x\sqrt{\operatorname{sech}(x)}}{21}\right)dx$$

```
Optimal(type 3, 31 leaves, 5 steps):
```

$$-\frac{4}{49\operatorname{sech}(x)^{7/2}}-\frac{20}{63\operatorname{sech}(x)^{3/2}}+\frac{2 x\sinh(x)}{7\operatorname{sech}(x)^{5/2}}+\frac{10 x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}$$

```
Result(type 8, 16 leaves):
```

$$\int\left(\frac{x}{\operatorname{sech}(x)^{7/2}}-\frac{5 x\sqrt{\operatorname{sech}(x)}}{21}\right)dx$$

```
Problem 6: Unable to integrate problem.
```

$$\int \left(\frac{x^2}{\operatorname{sech}(x)^{3/2}} - \frac{x^2 \sqrt{\operatorname{sech}(x)}}{3} \right) dx$$

Optimal(type 4, 63 leaves, 7 steps):

$$-\frac{8x}{9\operatorname{sech}(x)^{3/2}} + \frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}} + \frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}} - \frac{16 \mathrm{I} \sqrt{\cosh\left(\frac{x}{2}\right)^2}\, \operatorname{EllipticF}\left(\mathrm{I}\sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{\cosh(x)}\, \sqrt{\operatorname{sech}(x)}}{27\cosh\left(\frac{x}{2}\right)}$$

Result(type 8, 20 leaves):

$$\int \left(\frac{x^2}{\operatorname{sech}(x)^{3/2}} - \frac{x^2 \sqrt{\operatorname{sech}(x)}}{3} \right) dx$$

Test results for the 25 problems in "6.5.2 (e x)^m (a+b sech(c+d x^n))^p.txt"

Problem 1: Unable to integrate problem.

$$\int x^5 \left(a + b\operatorname{sech}(d x^2 + c)\right) dx$$

Optimal(type 4, 110 leaves, 10 steps):

$$\frac{a x^6}{6} + \frac{b x^4 \arctan\left(e^{d x^2 + c}\right)}{d} - \frac{\mathrm{I}\, b x^2 \operatorname{polylog}\left(2, -\mathrm{I}\, e^{d x^2 + c}\right)}{d^2} + \frac{\mathrm{I}\, b x^2 \operatorname{polylog}\left(2, \mathrm{I}\, e^{d x^2 + c}\right)}{d^2} + \frac{\mathrm{I}\, b \operatorname{polylog}\left(3, -\mathrm{I}\, e^{d x^2 + c}\right)}{d^3} - \frac{\mathrm{I}\, b \operatorname{polylog}\left(3, \mathrm{I}\, e^{d x^2 + c}\right)}{d^3}$$

Result(type 8, 37 leaves):

$$\frac{a x^6}{6} + \int \frac{2 e^{d x^2 + c} b x^5}{\left(e^{d x^2 + c}\right)^2 + 1} dx$$

Problem 2: Unable to integrate problem.

$$\int x^3 \left(a + b\operatorname{sech}(d x^2 + c)\right) dx$$

Optimal(type 4, 64 leaves, 8 steps):

$$\frac{a x^4}{4} + \frac{b x^2 \arctan\left(e^{d x^2 + c}\right)}{d} - \frac{\mathrm{I}\, b \operatorname{polylog}\left(2, -\mathrm{I}\, e^{d x^2 + c}\right)}{2 d^2} + \frac{\mathrm{I}\, b \operatorname{polylog}\left(2, \mathrm{I}\, e^{d x^2 + c}\right)}{2 d^2}$$

Result(type 8, 37 leaves):

$$\frac{a x^4}{4} + \int \frac{2 e^{d x^2 + c} b x^3}{\left(e^{d x^2 + c}\right)^2 + 1} dx$$

Problem 4: Unable to integrate problem.

$$\int x^3 \left(a + b\,\mathrm{sech}\left(d x^2 + c\right)\right)^2 dx$$

Optimal(type 4, 106 leaves, 10 steps):

$$\frac{a^2 x^4}{4} + \frac{2 a b x^2 \arctan\left(e^{d x^2 + c}\right)}{d} - \frac{b^2 \ln\left(\cosh\left(d x^2 + c\right)\right)}{2 d^2} - \frac{I a b\, \mathrm{polylog}\left(2, -I e^{d x^2 + c}\right)}{d^2} + \frac{I a b\, \mathrm{polylog}\left(2, I e^{d x^2 + c}\right)}{d^2} + \frac{b^2 x^2 \tanh\left(d x^2 + c\right)}{2 d}$$

Result(type 8, 74 leaves):

$$\frac{a^2 x^4}{4} - \frac{x^2 b^2}{d\left(\left(e^{d x^2 + c}\right)^2 + 1\right)} + \int \frac{2 b x \left(2 a d x^2 e^{d x^2 + c} + b\right)}{d\left(\left(e^{d x^2 + c}\right)^2 + 1\right)} dx$$

Problem 6: Unable to integrate problem.

$$\int \frac{x^5}{a + b\,\mathrm{sech}\left(d x^2 + c\right)} dx$$

Optimal(type 4, 313 leaves, 13 steps):

$$\frac{x^6}{6 a} - \frac{b x^4 \ln\left(1 + \frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{2 a d \sqrt{-a^2 + b^2}} + \frac{b x^4 \ln\left(1 + \frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{2 a d \sqrt{-a^2 + b^2}} - \frac{b x^2\, \mathrm{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a d^2 \sqrt{-a^2 + b^2}} + \frac{b x^2\, \mathrm{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a d^2 \sqrt{-a^2 + b^2}}$$
$$+ \frac{b\, \mathrm{polylog}\left(3, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a d^3 \sqrt{-a^2 + b^2}} - \frac{b\, \mathrm{polylog}\left(3, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a d^3 \sqrt{-a^2 + b^2}}$$

Result(type 8, 55 leaves):

$$\frac{x^6}{6 a} + \int -\frac{2 e^{d x^2 + c} b x^5}{a\left(a\left(e^{d x^2 + c}\right)^2 + 2 b e^{d x^2 + c} + a\right)} dx$$

Problem 8: Unable to integrate problem.

$$\int \frac{x^3}{a + b\,\mathrm{sech}\left(d x^2 + c\right)} dx$$

Optimal(type 4, 211 leaves, 11 steps):

$$\frac{x^4}{4 a} - \frac{b x^2 \ln\left(1 + \frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{2 a d \sqrt{-a^2 + b^2}} + \frac{b x^2 \ln\left(1 + \frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{2 a d \sqrt{-a^2 + b^2}} - \frac{b\, \mathrm{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{2 a d^2 \sqrt{-a^2 + b^2}} + \frac{b\, \mathrm{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{2 a d^2 \sqrt{-a^2 + b^2}}$$

Result(type 8, 55 leaves):

$$\frac{x^4}{4a} + \int -\frac{2 b x^3 e^{d x^2 + c}}{a \left(a \left(e^{d x^2 + c}\right)^2 + 2 b e^{d x^2 + c} + a\right)} \, dx$$

Problem 10: Unable to integrate problem.

$$\int \frac{x^5}{\left(a + b \operatorname{sech}(d x^2 + c)\right)^2} \, dx$$

Optimal(type 4, 912 leaves, 31 steps):

$$\frac{b^2 x^4}{2 a^2 (a^2 - b^2) d} + \frac{x^6}{6 a^2} - \frac{b^2 x^2 \ln\left(1 + \frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^2} + \frac{b^3 x^4 \ln\left(1 + \frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{2 a^2 (-a^2 + b^2)^{3/2} d} - \frac{b^2 x^2 \ln\left(1 + \frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^2}$$
$$- \frac{b^3 x^4 \ln\left(1 + \frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{2 a^2 (-a^2 + b^2)^{3/2} d} - \frac{b^2 \operatorname{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^3} + \frac{b^3 x^2 \operatorname{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} - \frac{b^2 \operatorname{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^3}$$
$$- \frac{b^3 x^2 \operatorname{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} - \frac{b^3 \operatorname{polylog}\left(3, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3} + \frac{b^3 \operatorname{polylog}\left(3, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3}$$
$$+ \frac{b^2 x^4 \sinh(d x^2 + c)}{2 a (a^2 - b^2) d \left(b + a \cosh(d x^2 + c)\right)} - \frac{b x^4 \ln\left(1 + \frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 d \sqrt{-a^2 + b^2}} + \frac{b x^4 \ln\left(1 + \frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 d \sqrt{-a^2 + b^2}} - \frac{2 b x^2 \operatorname{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 d^2 \sqrt{-a^2 + b^2}}$$
$$+ \frac{2 b x^2 \operatorname{polylog}\left(2, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 d^2 \sqrt{-a^2 + b^2}} + \frac{2 b \operatorname{polylog}\left(3, -\frac{a e^{d x^2 + c}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 d^3 \sqrt{-a^2 + b^2}} - \frac{2 b \operatorname{polylog}\left(3, -\frac{a e^{d x^2 + c}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 d^3 \sqrt{-a^2 + b^2}}$$

Result(type 8, 177 leaves):

$$\frac{x^6}{6 a^2} - \frac{b^2 x^4 \left(b e^{d x^2 + c} + a\right)}{a^2 (a^2 - b^2) d \left(a \left(e^{d x^2 + c}\right)^2 + 2 b e^{d x^2 + c} + a\right)} + \int -\frac{2 b x^3 \left(2 a^2 d x^2 e^{d x^2 + c} - b^2 d x^2 e^{d x^2 + c} - 2 b^2 e^{d x^2 + c} - 2 a b\right)}{a^2 (a^2 - b^2) d \left(a \left(e^{d x^2 + c}\right)^2 + 2 b e^{d x^2 + c} + a\right)} \, dx$$

Problem 13: Unable to integrate problem.

$$\int x \left(a + b \operatorname{sech}\left(c + d \sqrt{x}\right)\right)^2 dx$$

Optimal(type 4, 264 leaves, 18 steps):

$$\frac{2 b^2 x^{3/2}}{d}+\frac{a^2 x^2}{2}+\frac{8 a b x^{3/2} \arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{6 b^2 x \ln\left(1+e^{2c+2d\sqrt{x}}\right)}{d^2}-\frac{12 I a b x \,\mathrm{polylog}\left(2,-I e^{c+d\sqrt{x}}\right)}{d^2}+\frac{12 I a b x \,\mathrm{polylog}\left(2,I e^{c+d\sqrt{x}}\right)}{d^2}$$
$$+\frac{3 b^2 \mathrm{polylog}\left(3,-e^{2c+2d\sqrt{x}}\right)}{d^4}-\frac{24 I a b \,\mathrm{polylog}\left(4,-I e^{c+d\sqrt{x}}\right)}{d^4}+\frac{24 I a b \,\mathrm{polylog}\left(4,I e^{c+d\sqrt{x}}\right)}{d^4}-\frac{6 b^2 \mathrm{polylog}\left(2,-e^{2c+2d\sqrt{x}}\right)\sqrt{x}}{d^3}$$
$$+\frac{24 I a b \,\mathrm{polylog}\left(3,-I e^{c+d\sqrt{x}}\right)\sqrt{x}}{d^3}-\frac{24 I a b \,\mathrm{polylog}\left(3,I e^{c+d\sqrt{x}}\right)\sqrt{x}}{d^3}+\frac{2 b^2 x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d}$$

Result(type 8, 18 leaves):

$$\int x\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2 dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{x^3}{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\, dx$$

Optimal(type 4, 2503 leaves, 61 steps):

$$\frac{10080 b^2 \mathrm{polylog}\left(7,-\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^8}+\frac{10080 b^2 \mathrm{polylog}\left(7,-\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^8}+\frac{10080 b^3 \mathrm{polylog}\left(8,-\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^8}$$
$$-\frac{10080 b^3 \mathrm{polylog}\left(8,-\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^8}-\frac{20160 b \,\mathrm{polylog}\left(8,-\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 d^8\sqrt{-a^2+b^2}}+\frac{20160 b \,\mathrm{polylog}\left(8,-\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 d^8\sqrt{-a^2+b^2}}+\frac{2 b^2 x^{7/2}}{a^2\left(a^2-b^2\right)d}$$
$$+\frac{2 b^3 x^{7/2}\ln\left(1+\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}-\frac{14 b^2 x^3\ln\left(1+\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}-\frac{2 b^3 x^{7/2}\ln\left(1+\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$
$$-\frac{84 b^2 x^{5/2}\mathrm{polylog}\left(2,-\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}+\frac{14 b^3 x^3\mathrm{polylog}\left(2,-\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}-\frac{84 b^2 x^{5/2}\mathrm{polylog}\left(2,-\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{14 b^3 x^3\mathrm{polylog}\left(2,-\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}+\frac{420 b^2 x^2\mathrm{polylog}\left(3,-\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}-\frac{84 b^3 x^{5/2}\mathrm{polylog}\left(3,-\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

$$+\frac{420\,b^2 x^2 \operatorname{polylog}\left(3,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}+\frac{84\,b^3 x^{5/2} \operatorname{polylog}\left(3,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}-\frac{1680\,b^2 x^{3/2} \operatorname{polylog}\left(4,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^5}$$

$$+\frac{420\,b^3 x^2 \operatorname{polylog}\left(4,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^4}-\frac{1680\,b^2 x^{3/2} \operatorname{polylog}\left(4,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^5}-\frac{420\,b^3 x^2 \operatorname{polylog}\left(4,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^4}$$

$$+\frac{5040\,b^2 x \operatorname{polylog}\left(5,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^6}-\frac{1680\,b^3 x^{3/2} \operatorname{polylog}\left(5,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^5}+\frac{5040\,b^2 x \operatorname{polylog}\left(5,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^6}$$

$$+\frac{1680\,b^3 x^{3/2} \operatorname{polylog}\left(5,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^5}+\frac{5040\,b^3 x \operatorname{polylog}\left(6,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^6}-\frac{5040\,b^3 x \operatorname{polylog}\left(6,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^6}$$

$$-\frac{4\,b\,x^{7/2} \ln\left(1+\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d\sqrt{-a^2+b^2}}+\frac{4\,b\,x^{7/2} \ln\left(1+\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d\sqrt{-a^2+b^2}}-\frac{28\,b\,x^3 \operatorname{polylog}\left(2,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d^2\sqrt{-a^2+b^2}}$$

$$+\frac{28\,b\,x^3 \operatorname{polylog}\left(2,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d^2\sqrt{-a^2+b^2}}+\frac{168\,b\,x^{5/2} \operatorname{polylog}\left(3,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d^3\sqrt{-a^2+b^2}}-\frac{168\,b\,x^{5/2} \operatorname{polylog}\left(3,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d^3\sqrt{-a^2+b^2}}$$

$$-\frac{840\,b\,x^2 \operatorname{polylog}\left(4,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d^4\sqrt{-a^2+b^2}}+\frac{840\,b\,x^2 \operatorname{polylog}\left(4,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d^4\sqrt{-a^2+b^2}}+\frac{3360\,b\,x^{3/2} \operatorname{polylog}\left(5,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d^5\sqrt{-a^2+b^2}}$$

$$-\frac{3360\,b\,x^{3/2} \operatorname{polylog}\left(5,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d^5\sqrt{-a^2+b^2}}-\frac{10080\,b\,x \operatorname{polylog}\left(6,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d^6\sqrt{-a^2+b^2}}+\frac{10080\,b\,x \operatorname{polylog}\left(6,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d^6\sqrt{-a^2+b^2}}$$

$$-\frac{10080\,b^2 \operatorname{polylog}\left(6,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2\left(a^2-b^2\right)d^7}-\frac{10080\,b^2 \operatorname{polylog}\left(6,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2\left(a^2-b^2\right)d^7}-\frac{10080\,b^3 \operatorname{polylog}\left(7,-\frac{a\,\mathrm{e}^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2\left(-a^2+b^2\right)^{3/2}d^7}$$

$$+\frac{10080\, b^3\, \mathrm{polylog}\left(7, -\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2\left(-a^2+b^2\right)^{3/2} d^7}+\frac{20160\, b\, \mathrm{polylog}\left(7, -\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 d^7\sqrt{-a^2+b^2}}-\frac{20160\, b\, \mathrm{polylog}\left(7, -\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 d^7\sqrt{-a^2+b^2}}$$

$$-\frac{14\, b^2 x^3 \ln\left(1+\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right) d^2}+\frac{x^4}{4\, a^2}+\frac{2\, b^2 x^{7/2} \sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right) d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$

Result(type 8, 20 leaves):

$$\int \frac{x^3}{\left(a+b\, \mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\, dx$$

Problem 17: Unable to integrate problem.

$$\int \frac{x}{\left(a+b\, \mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\, dx$$

Optimal(type 4, 1223 leaves, 37 steps):

$$\frac{12\, b^2\, \mathrm{polylog}\left(3, -\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right) d^4}+\frac{12\, b^2\, \mathrm{polylog}\left(3, -\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right) d^4}+\frac{12\, b^3\, \mathrm{polylog}\left(4, -\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2} d^4}$$

$$-\frac{12\, b^3\, \mathrm{polylog}\left(4, -\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2} d^4}-\frac{24\, b\, \mathrm{polylog}\left(4, -\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 d^4\sqrt{-a^2+b^2}}+\frac{24\, b\, \mathrm{polylog}\left(4, -\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 d^4\sqrt{-a^2+b^2}}+\frac{2\, b^2 x^{3/2}}{a^2\left(a^2-b^2\right) d}$$

$$-\frac{6\, b^2 x \ln\left(1+\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right) d^2}+\frac{2\, b^3 x^{3/2} \ln\left(1+\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2} d}-\frac{6\, b^2 x \ln\left(1+\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right) d^2}-\frac{2\, b^3 x^{3/2} \ln\left(1+\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2} d}$$

$$+\frac{6\, b^3 x\, \mathrm{polylog}\left(2, -\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2} d^2}-\frac{6\, b^3 x\, \mathrm{polylog}\left(2, -\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2} d^2}-\frac{4\, b\, x^{3/2} \ln\left(1+\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 d\sqrt{-a^2+b^2}}$$

$$+\frac{4\, b\, x^{3/2} \ln\left(1+\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 d\sqrt{-a^2+b^2}}-\frac{12\, b\, x\, \mathrm{polylog}\left(2, -\frac{a\, e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 d^2\sqrt{-a^2+b^2}}+\frac{12\, b\, x\, \mathrm{polylog}\left(2, -\frac{a\, e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 d^2\sqrt{-a^2+b^2}}$$

$$-\frac{12 b^2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 (a^2-b^2) d^3} - \frac{12 b^2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 (a^2-b^2) d^3} - \frac{12 b^3 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 (-a^2+b^2)^{3/2} d^3}$$

$$+\frac{12 b^3 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 (-a^2+b^2)^{3/2} d^3} + \frac{24 b \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 d^3 \sqrt{-a^2+b^2}} - \frac{24 b \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a^2 d^3 \sqrt{-a^2+b^2}} + \frac{x^2}{2 a^2}$$

$$+\frac{2 b^2 x^{3/2} \sinh(c+d\sqrt{x})}{a (a^2-b^2) d \left(b + a\cosh(c+d\sqrt{x})\right)}$$

Result(type 8, 18 leaves):

$$\int \frac{x}{\left(a+b\operatorname{sech}(c+d\sqrt{x})\right)^2}\, dx$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2n}}{a+b\operatorname{sech}(c+d x^n)}\, dx$$

Optimal(type 4, 289 leaves, 12 steps):

$$\frac{(e x)^{2n}}{2 a e n} - \frac{b (e x)^{2n} \ln\left(1+\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right)}{a d e n x^n \sqrt{-a^2+b^2}} + \frac{b (e x)^{2n} \ln\left(1+\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right)}{a d e n x^n \sqrt{-a^2+b^2}} - \frac{b (e x)^{2n} \operatorname{polylog}\left(2, -\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right)}{a d^2 e n x^{2n} \sqrt{-a^2+b^2}}$$

$$+\frac{b (e x)^{2n} \operatorname{polylog}\left(2, -\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right)}{a d^2 e n x^{2n} \sqrt{-a^2+b^2}}$$

Result(type 4, 586 leaves):

$$\frac{x\, e^{\frac{(-1+2n)\left(-I\pi \operatorname{csgn}(I e x)^3 + I\pi \operatorname{csgn}(I e x)^2 \operatorname{csgn}(I e) + I\pi \operatorname{csgn}(I e x)^2 \operatorname{csgn}(I x) - I\pi \operatorname{csgn}(I e x)\operatorname{csgn}(I e)\operatorname{csgn}(I x) + 2\ln(e) + 2\ln(x)\right)}{2}}}{2 a n}$$

$$-\frac{1}{a e n d^2}\Bigg(2 b\, e^{-I\pi n \operatorname{csgn}(I e)\operatorname{csgn}(I x)\operatorname{csgn}(I e x)}\, e^{I\pi n \operatorname{csgn}(I e)\operatorname{csgn}(I e x)^2}\, e^{I\pi n \operatorname{csgn}(I x)\operatorname{csgn}(I e x)^2}\, e^{-I\pi n \operatorname{csgn}(I e x)^3}\, e^{\frac{I}{2}\pi \operatorname{csgn}(I e)\operatorname{csgn}(I x)\operatorname{csgn}(I e x)}\, e^{-\frac{I}{2}\pi \operatorname{csgn}(I e)\operatorname{csgn}(I e x)^2}$$

$$e^{-\frac{I}{2}\pi \operatorname{csgn}(I x)\operatorname{csgn}(I e x)^2} e^{\frac{I}{2}\pi \operatorname{csgn}(I e x)^3} (e^n)^2 e^c \left(\frac{d x^n \left(\ln\left(\frac{a e^{2c+d x^n}+e^c b-\sqrt{e^{2c}b^2-a^2 e^{2c}}}{e^c b-\sqrt{e^{2c}b^2-a^2e^{2c}}}\right)-\ln\left(\frac{a e^{2c+d x^n}+e^c b+\sqrt{e^{2c}b^2-a^2 e^{2c}}}{e^c b+\sqrt{e^{2c}b^2-a^2e^{2c}}}\right)\right)}{2\sqrt{e^{2c}b^2-a^2e^{2c}}}\right.$$
$$\left.+\frac{\operatorname{dilog}\left(\frac{a e^{2c+d x^n}+e^c b-\sqrt{e^{2c}b^2-a^2 e^{2c}}}{e^c b-\sqrt{e^{2c}b^2-a^2e^{2c}}}\right)-\operatorname{dilog}\left(\frac{a e^{2c+d x^n}+e^c b+\sqrt{e^{2c}b^2-a^2 e^{2c}}}{e^c b+\sqrt{e^{2c}b^2-a^2e^{2c}}}\right)}{2\sqrt{e^{2c}b^2-a^2e^{2c}}}\right)\Bigg)$$

Problem 25: Unable to integrate problem.

$$\int \frac{(e x)^{-1+3n}}{a+b\operatorname{sech}(c+d x^n)}\, dx$$

Optimal(type 4, 428 leaves, 14 steps):

$$\frac{(e x)^{3n}}{3 a e n}-\frac{b (e x)^{3n}\ln\left(1+\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right)}{a d e n x^n\sqrt{-a^2+b^2}}+\frac{b (e x)^{3n}\ln\left(1+\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right)}{a d e n x^n\sqrt{-a^2+b^2}}-\frac{2 b (e x)^{3n}\operatorname{polylog}\left(2,-\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right)}{a d^2 e n x^{2n}\sqrt{-a^2+b^2}}$$
$$+\frac{2 b (e x)^{3n}\operatorname{polylog}\left(2,-\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right)}{a d^2 e n x^{2n}\sqrt{-a^2+b^2}}+\frac{2 b (e x)^{3n}\operatorname{polylog}\left(3,-\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right)}{a d^3 e n x^{3n}\sqrt{-a^2+b^2}}-\frac{2 b (e x)^{3n}\operatorname{polylog}\left(3,-\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right)}{a d^3 e n x^{3n}\sqrt{-a^2+b^2}}$$

Result(type 8, 159 leaves):

$$\frac{x\, e^{(-1+3n)\left(\ln(e)+\ln(x)-\frac{I\pi \operatorname{csgn}(I e x)(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I e))(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I x))}{2}\right)}}{3 a n}+\int -\frac{2 b\, e^{(-1+3n)\left(\ln(e)+\ln(x)-\frac{I\pi \operatorname{csgn}(I e x)(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I e))(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I x))}{2}\right)} e^{c+d e^{n\ln(x)}}}{a\left(a\left(e^{c+d e^{n\ln(x)}}\right)^2+2 b e^{c+d e^{n\ln(x)}}+a\right)}\, dx$$

Test results for the 52 problems in "6.5.3 Hyperbolic secant functions.txt"

Problem 5: Unable to integrate problem.

$$\int (b\operatorname{sech}(d x+c))^{7/2}\, dx$$

Optimal(type 4, 114 leaves, 4 steps):

$$\frac{2 b (b \operatorname{sech}(d x+c))^{5/2} \sinh(d x+c)}{5 d}+\frac{6 I b^4 \sqrt{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}\, \operatorname{EllipticE}\left(I \sinh\left(\frac{d x}{2}+\frac{c}{2}\right), \sqrt{2}\right)}{5 \cosh\left(\frac{d x}{2}+\frac{c}{2}\right) d \sqrt{\cosh(d x+c)}\, \sqrt{b \operatorname{sech}(d x+c)}}+\frac{6 b^3 \sinh(d x+c) \sqrt{b \operatorname{sech}(d x+c)}}{5 d}$$

Result(type 8, 12 leaves):

$$\int (b \operatorname{sech}(d x+c))^{7/2}\, dx$$

Problem 6: Unable to integrate problem.

$$\int (b \operatorname{sech}(d x+c))^{5/2}\, dx$$

Optimal(type 4, 90 leaves, 3 steps):

$$\frac{2 b (b \operatorname{sech}(d x+c))^{3/2} \sinh(d x+c)}{3 d}-\frac{2 I b^2 \sqrt{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}\, \operatorname{EllipticF}\left(I \sinh\left(\frac{d x}{2}+\frac{c}{2}\right), \sqrt{2}\right) \sqrt{\cosh(d x+c)}\, \sqrt{b \operatorname{sech}(d x+c)}}{3 \cosh\left(\frac{d x}{2}+\frac{c}{2}\right) d}$$

Result(type 8, 12 leaves):

$$\int (b \operatorname{sech}(d x+c))^{5/2}\, dx$$

Problem 7: Unable to integrate problem.

$$\int (b \operatorname{sech}(d x+c))^{3/2}\, dx$$

Optimal(type 4, 90 leaves, 3 steps):

$$\frac{2 I b^2 \sqrt{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}\, \operatorname{EllipticE}\left(I \sinh\left(\frac{d x}{2}+\frac{c}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right) d \sqrt{\cosh(d x+c)}\, \sqrt{b \operatorname{sech}(d x+c)}}+\frac{2 b \sinh(d x+c) \sqrt{b \operatorname{sech}(d x+c)}}{d}$$

Result(type 8, 12 leaves):

$$\int (b \operatorname{sech}(d x+c))^{3/2}\, dx$$

Problem 8: Unable to integrate problem.

$$\int \sqrt{b \operatorname{sech}(d x+c)}\, dx$$

Optimal(type 4, 64 leaves, 2 steps):

$$-\frac{-2\,\mathrm{I}\sqrt{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\ \mathrm{EllipticF}\left(\mathrm{I}\sinh\left(\frac{d x}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\cosh(d x+c)}\ \sqrt{b\,\mathrm{sech}(d x+c)}}{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right) d}$$

Result(type 8, 12 leaves):

$$\int\sqrt{b\,\mathrm{sech}(d x+c)}\ \mathrm{d}x$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{b\,\mathrm{sech}(d x+c)}}\ \mathrm{d}x$$

Optimal(type 4, 64 leaves, 2 steps):

$$\frac{-2\,\mathrm{I}\sqrt{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\ \mathrm{EllipticE}\left(\mathrm{I}\sinh\left(\frac{d x}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cosh\left(\frac{d x}{2}+\frac{c}{2}\right) d\sqrt{\cosh(d x+c)}\ \sqrt{b\,\mathrm{sech}(d x+c)}}$$

Result(type 4, 243 leaves):

$$\frac{\sqrt{2}}{d\sqrt{\frac{b\,\mathrm{e}^{d x+c}}{(\mathrm{e}^{d x+c})^{2}+1}}}+\frac{1}{d\sqrt{\frac{b\,\mathrm{e}^{d x+c}}{(\mathrm{e}^{d x+c})^{2}+1}}\ \left((\mathrm{e}^{d x+c})^{2}+1\right)}\left(\left(-\frac{2\left((\mathrm{e}^{d x+c})^{2} b+b\right)}{b\sqrt{\mathrm{e}^{d x+c}\left((\mathrm{e}^{d x+c})^{2} b+b\right)}}\right.\right.$$

$$\left.+\frac{\mathrm{I}\sqrt{-\mathrm{I}\left(\mathrm{e}^{d x+c}+\mathrm{I}\right)}\ \sqrt{2}\ \sqrt{\mathrm{I}\left(\mathrm{e}^{d x+c}-\mathrm{I}\right)}\ \sqrt{\mathrm{I}\,\mathrm{e}^{d x+c}}\ \left(-2\,\mathrm{I}\,\mathrm{EllipticE}\left(\sqrt{-\mathrm{I}\left(\mathrm{e}^{d x+c}+\mathrm{I}\right)},\frac{\sqrt{2}}{2}\right)+\mathrm{I}\,\mathrm{EllipticF}\left(\sqrt{-\mathrm{I}\left(\mathrm{e}^{d x+c}+\mathrm{I}\right)},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{b\left(\mathrm{e}^{d x+c}\right)^{3}+b\,\mathrm{e}^{d x+c}}}\right)$$

$$\left.\sqrt{2}\ \sqrt{b\,\mathrm{e}^{d x+c}\left((\mathrm{e}^{d x+c})^{2}+1\right)}\right)$$

Problem 10: Unable to integrate problem.

$$\int(b\,\mathrm{sech}(d x+c))^{n}\ \mathrm{d}x$$

Optimal(type 5, 65 leaves, 2 steps):

$$-\frac{b\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{n}{2}\right],\cosh(d x+c)^{2}\right)(b\,\mathrm{sech}(d x+c))^{-1+n}\sinh(d x+c)}{d(1-n)\sqrt{-\sinh(d x+c)^{2}}}$$

Result(type 8, 12 leaves):

$$\int (b \operatorname{sech}(d x + c))^n \, dx$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(\operatorname{sech}(b x + a)^2\right)^{3/2}} \, dx$$

Optimal(type 3, 43 leaves, 3 steps):

$$\frac{\tanh(b x + a)}{3 b \left(\operatorname{sech}(b x + a)^2\right)^{3/2}} + \frac{2 \tanh(b x + a)}{3 b \sqrt{\operatorname{sech}(b x + a)^2}}$$

Result(type 3, 200 leaves):

$$\frac{e^{4 b x + 4 a}}{24 \left(e^{2 b x + 2 a} + 1\right) \sqrt{\frac{e^{2 b x + 2 a}}{\left(e^{2 b x + 2 a} + 1\right)^2}}\, b} + \frac{3 e^{2 b x + 2 a}}{8 \left(e^{2 b x + 2 a} + 1\right) \sqrt{\frac{e^{2 b x + 2 a}}{\left(e^{2 b x + 2 a} + 1\right)^2}}\, b} - \frac{3}{8 \left(e^{2 b x + 2 a} + 1\right) \sqrt{\frac{e^{2 b x + 2 a}}{\left(e^{2 b x + 2 a} + 1\right)^2}}\, b} - \frac{e^{-2 b x - 2 a}}{24 \left(e^{2 b x + 2 a} + 1\right) \sqrt{\frac{e^{2 b x + 2 a}}{\left(e^{2 b x + 2 a} + 1\right)^2}}\, b}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a \operatorname{sech}(x)^2}} \, dx$$

Optimal(type 3, 11 leaves, 2 steps):

$$\frac{\tanh(x)}{\sqrt{a \operatorname{sech}(x)^2}}$$

Result(type 3, 57 leaves):

$$\frac{e^{2 x}}{2 \sqrt{\frac{a e^{2 x}}{\left(e^{2 x} + 1\right)^2}} \left(e^{2 x} + 1\right)} - \frac{1}{2 \sqrt{\frac{a e^{2 x}}{\left(e^{2 x} + 1\right)^2}} \left(e^{2 x} + 1\right)}$$

Problem 13: Unable to integrate problem.

$$\int \left(a \operatorname{sech}(x)^3\right)^{5/2} \, dx$$

Optimal(type 4, 114 leaves, 7 steps):

$$\frac{154\,I\,a^2\cosh(x)^{3/2}\sqrt{\cosh\left(\frac{x}{2}\right)^2}\,\mathrm{EllipticE}\left(I\sinh\left(\frac{x}{2}\right),\sqrt{2}\right)\sqrt{a\,\mathrm{sech}(x)^3}}{195\cosh\left(\frac{x}{2}\right)}+\frac{154\,a^2\cosh(x)\sinh(x)\sqrt{a\,\mathrm{sech}(x)^3}}{195}+\frac{154\,a^2\sqrt{a\,\mathrm{sech}(x)^3}\,\tanh(x)}{585}$$

$$+\frac{22\,a^2\,\mathrm{sech}(x)^2\sqrt{a\,\mathrm{sech}(x)^3}\,\tanh(x)}{117}+\frac{2\,a^2\,\mathrm{sech}(x)^4\sqrt{a\,\mathrm{sech}(x)^3}\,\tanh(x)}{13}$$

Result(type 8, 10 leaves):

$$\int\left(a\,\mathrm{sech}(x)^3\right)^{5/2}\,dx$$

Problem 14: Unable to integrate problem.

$$\int\sqrt{a\,\mathrm{sech}(x)^3}\,dx$$

Optimal(type 4, 55 leaves, 4 steps):

$$\frac{2\,I\cosh(x)^{3/2}\sqrt{\cosh\left(\frac{x}{2}\right)^2}\,\mathrm{EllipticE}\left(I\sinh\left(\frac{x}{2}\right),\sqrt{2}\right)\sqrt{a\,\mathrm{sech}(x)^3}}{\cosh\left(\frac{x}{2}\right)}+2\cosh(x)\sinh(x)\sqrt{a\,\mathrm{sech}(x)^3}$$

Result(type 8, 10 leaves):

$$\int\sqrt{a\,\mathrm{sech}(x)^3}\,dx$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{a\,\mathrm{sech}(x)^4}}\,dx$$

Optimal(type 3, 28 leaves, 3 steps):

$$\frac{x\,\mathrm{sech}(x)^2}{2\sqrt{a\,\mathrm{sech}(x)^4}}+\frac{\tanh(x)}{2\sqrt{a\,\mathrm{sech}(x)^4}}$$

Result(type 3, 88 leaves):

$$\frac{e^{2x}x}{2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}+1\right)^4}}\left(e^{2x}+1\right)^2}+\frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}+1\right)^4}}\left(e^{2x}+1\right)^2}-\frac{1}{8\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}+1\right)^4}}\left(e^{2x}+1\right)^2}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)^2}{a + a\,\mathrm{sech}(x)}\, dx$$

Optimal(type 3, 37 leaves, 5 steps):

$$\frac{3x}{2a} - \frac{2\sinh(x)}{a} + \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh(x)\sinh(x)}{a + a\,\mathrm{sech}(x)}$$

Result(type 3, 86 leaves):

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a} - \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a} + \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$
$$- \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}$$

Problem 24: Unable to integrate problem.

$$\int \sqrt{a + a\,\mathrm{sech}(dx+c)}\, dx$$

Optimal(type 3, 31 leaves, 2 steps):

$$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a}\,\tanh(dx+c)}{\sqrt{a + a\,\mathrm{sech}(dx+c)}}\right)\sqrt{a}}{d}$$

Result(type 8, 14 leaves):

$$\int \sqrt{a + a\,\mathrm{sech}(dx+c)}\, dx$$

Problem 25: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + a\,\mathrm{sech}(dx+c)}}\, dx$$

Optimal(type 3, 70 leaves, 5 steps):

$$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a}\,\tanh(dx+c)}{\sqrt{a + a\,\mathrm{sech}(dx+c)}}\right)}{d\sqrt{a}} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{a}\,\tanh(dx+c)\sqrt{2}}{2\sqrt{a + a\,\mathrm{sech}(dx+c)}}\right)\sqrt{2}}{d\sqrt{a}}$$

Result(type 8, 14 leaves):

$$\int \frac{1}{\sqrt{a + a\,\mathrm{sech}(dx+c)}}\, dx$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b\,\mathrm{sech}(dx+c))^2}\,dx$$

Optimal(type 3, 100 leaves, 5 steps):

$$\frac{x}{a^2}-\frac{2b\left(2a^2-b^2\right)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b^2\tanh(dx+c)}{a\left(a^2-b^2\right)d\,(a+b\,\mathrm{sech}(dx+c))}$$

Result(type 3, 220 leaves):

$$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}+\frac{2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a\left(a^2-b^2\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}$$
$$-\frac{4b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d(a+b)(a-b)\sqrt{(a+b)(a-b)}}+\frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\,a^2(a+b)(a-b)\sqrt{(a+b)(a-b)}}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^7}{a+b\,\mathrm{sech}(x)}\,dx$$

Optimal(type 3, 113 leaves, 3 steps):

$$\frac{\ln(\cosh(x))}{a}-\frac{\left(a^2-b^2\right)^3\ln(a+b\,\mathrm{sech}(x))}{a\,b^6}+\frac{\left(a^4-3a^2b^2+3b^4\right)\mathrm{sech}(x)}{b^5}-\frac{a\left(a^2-3b^2\right)\mathrm{sech}(x)^2}{2b^4}+\frac{\left(a^2-3b^2\right)\mathrm{sech}(x)^3}{3b^3}-\frac{a\,\mathrm{sech}(x)^4}{4b^2}$$
$$+\frac{\mathrm{sech}(x)^5}{5b}$$

Result(type 3, 414 leaves):

$$\frac{32}{5b\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^5}+\frac{8a^2}{3b^3\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^3}+\frac{8a}{b^2\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^3}+\frac{8}{b\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^3}-\frac{16}{b\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^4}-\frac{4a}{b^2\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^4}$$
$$+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)a^5}{b^6}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)a^3}{b^4}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)a}{b^2}-\frac{2a^3}{b^4\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}-\frac{4a^2}{b^3\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}$$

$$+\frac{4}{b\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}+\frac{2a^4}{b^5\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}+\frac{2a^3}{b^4\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}-\frac{4a^2}{b^3\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}-\frac{4a}{b^2\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}+\frac{2}{b\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}$$

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{a^5\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{b^6}+\frac{3a^3\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{b^4}$$

$$-\frac{3a\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{b^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{a}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{\tanh(x)^3}{a+b\,\mathrm{sech}(x)}\,dx$$

Optimal(type 3, 35 leaves, 3 steps):

$$\frac{\ln(\cosh(x))}{a}+\frac{\left(1-\frac{a^2}{b^2}\right)\ln(a+b\,\mathrm{sech}(x))}{a}+\frac{\mathrm{sech}(x)}{b}$$

Result(type 3, 106 leaves):

$$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)a}{b^2}+\frac{2}{b\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{b^2}$$

$$+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{a}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int\frac{\tanh(x)^2}{a+b\,\mathrm{sech}(x)}\,dx$$

Optimal(type 3, 52 leaves, 7 steps):

$$\frac{x}{a}-\frac{\arctan(\sinh(x))}{b}+\frac{2\arctan\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)\sqrt{a-b}\,\sqrt{a+b}}{a\,b}$$

Result(type 3, 112 leaves):

$$-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2\,a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)\,(a-b)}}\right)}{b\sqrt{(a+b)\,(a-b)}}-\frac{2\,b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)\,(a-b)}}\right)}{a\sqrt{(a+b)\,(a-b)}}$$

Problem 38: Unable to integrate problem.

$$\int\sqrt{a+b\,\mathrm{sech}(d\,x+c)}\,\tanh(d\,x+c)^3\,dx$$

Optimal(type 3, 84 leaves, 5 steps):

$$-\frac{2\,a\,(a+b\,\mathrm{sech}(d\,x+c))^{3/2}}{3\,b^2\,d}+\frac{2\,(a+b\,\mathrm{sech}(d\,x+c))^{5/2}}{5\,b^2\,d}+\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(d\,x+c)}}{\sqrt{a}}\right)\sqrt{a}}{d}-\frac{2\sqrt{a+b\,\mathrm{sech}(d\,x+c)}}{d}$$

Result(type 8, 23 leaves):

$$\int\sqrt{a+b\,\mathrm{sech}(d\,x+c)}\,\tanh(d\,x+c)^3\,dx$$

Problem 39: Unable to integrate problem.

$$\int\sqrt{a+b\,\mathrm{sech}(d\,x+c)}\,dx$$

Optimal(type 4, 116 leaves, 1 step):

$$\frac{2\coth(d\,x+c)\,\mathrm{EllipticPi}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\,\mathrm{sech}(d\,x+c)}},\frac{a}{a+b},\sqrt{\frac{a-b}{a+b}}\right)(a+b\,\mathrm{sech}(d\,x+c))\sqrt{-\frac{b\,(1-\mathrm{sech}(d\,x+c))}{a+b\,\mathrm{sech}(d\,x+c)}}\sqrt{\frac{b\,(1+\mathrm{sech}(d\,x+c))}{a+b\,\mathrm{sech}(d\,x+c)}}}{d\sqrt{a+b}}$$

Result(type 8, 14 leaves):

$$\int\sqrt{a+b\,\mathrm{sech}(d\,x+c)}\,dx$$

Problem 40: Unable to integrate problem.

$$\int\frac{\tanh(d\,x+c)^5}{\sqrt{a+b\,\mathrm{sech}(d\,x+c)}}\,dx$$

Optimal(type 3, 128 leaves, 5 steps):

$$-\frac{2\left(3\,a^2-2\,b^2\right)(a+b\,\mathrm{sech}(d\,x+c))^{3/2}}{3\,b^4\,d}+\frac{6\,a\,(a+b\,\mathrm{sech}(d\,x+c))^{5/2}}{5\,b^4\,d}-\frac{2\,(a+b\,\mathrm{sech}(d\,x+c))^{7/2}}{7\,b^4\,d}+\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(d\,x+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$$

$$+\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\operatorname{sech}(dx+c)}}{b^4 d}$$

Result(type 8, 23 leaves):

$$\int \frac{\tanh(dx+c)^5}{\sqrt{a+b\operatorname{sech}(dx+c)}}\,dx$$

Problem 41: Unable to integrate problem.

$$\int \frac{\tanh(dx+c)^5}{(a+b\operatorname{sech}(dx+c))^{3/2}}\,dx$$

Optimal(type 3, 132 leaves, 5 steps):

$$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2a(a+b\operatorname{sech}(dx+c))^{3/2}}{b^4 d}-\frac{2(a+b\operatorname{sech}(dx+c))^{5/2}}{5b^4 d}-\frac{2\left(a^2-b^2\right)^2}{a b^4 d\sqrt{a+b\operatorname{sech}(dx+c)}}$$

$$-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\operatorname{sech}(dx+c)}}{b^4 d}$$

Result(type 8, 23 leaves):

$$\int \frac{\tanh(dx+c)^5}{(a+b\operatorname{sech}(dx+c))^{3/2}}\,dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{\tanh(dx+c)^4}{(a+b\operatorname{sech}(dx+c))^{3/2}}\,dx$$

Optimal(type 4, 830 leaves, 17 steps):

$$-\frac{2\coth(dx+c)\,\mathrm{EllipticE}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{a d\sqrt{a+b}}$$

$$+\frac{4a\coth(dx+c)\,\mathrm{EllipticE}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{b^2 d\sqrt{a+b}}$$

$$-\frac{2a\left(8a^2-5b^2\right)\coth(dx+c)\,\mathrm{EllipticE}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{3b^4 d\sqrt{a+b}}$$

$$+\frac{2\coth(dx+c)\operatorname{EllipticF}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{a\,d\sqrt{a+b}}$$

$$+\frac{4\coth(dx+c)\operatorname{EllipticF}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{b\,d\sqrt{a+b}}$$

$$-\frac{2(2a+b)(4a+b)\coth(dx+c)\operatorname{EllipticF}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{3b^3d\sqrt{a+b}}$$

$$+\frac{2\coth(dx+c)\operatorname{EllipticPi}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\frac{a+b}{a},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{a+b}\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{a^2d}$$

$$-\frac{4a\tanh(dx+c)}{(a^2-b^2)d\sqrt{a+b\operatorname{sech}(dx+c)}}+\frac{2b^2\tanh(dx+c)}{a(a^2-b^2)d\sqrt{a+b\operatorname{sech}(dx+c)}}-\frac{2a^2\operatorname{sech}(dx+c)\tanh(dx+c)}{b(a^2-b^2)d\sqrt{a+b\operatorname{sech}(dx+c)}}$$

$$+\frac{2(4a^2-b^2)\sqrt{a+b\operatorname{sech}(dx+c)}\tanh(dx+c)}{3b^2(a^2-b^2)d}$$

Result(type 8, 23 leaves):

$$\int\frac{\tanh(dx+c)^4}{(a+b\operatorname{sech}(dx+c))^{3/2}}\,dx$$

Problem 43: Unable to integrate problem.

$$\int\frac{\tanh(dx+c)^2}{(a+b\operatorname{sech}(dx+c))^{3/2}}\,dx$$

Optimal(type 4, 315 leaves, 7 steps):

$$\frac{2(a-b)\coth(dx+c)\operatorname{EllipticE}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{a+b}\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{a\,b^2d}$$

$$+\frac{2\coth(dx+c)\operatorname{EllipticF}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{a+b}\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{a\,b\,d}$$

$$+\frac{2\coth(dx+c)\operatorname{EllipticPi}\left(\frac{\sqrt{a+b\operatorname{sech}(dx+c)}}{\sqrt{a+b}},\frac{a+b}{a},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{a+b}\sqrt{\frac{b(1-\operatorname{sech}(dx+c))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(dx+c))}{a-b}}}{a^2d}$$

$$-\frac{2\tanh(dx+c)}{a\,d\sqrt{a+b\,\mathrm{sech}(dx+c)}}$$

Result(type 8, 23 leaves):

$$\int \frac{\tanh(dx+c)^2}{(a+b\,\mathrm{sech}(dx+c))^{3/2}}\,dx$$

Problem 47: Unable to integrate problem.

$$\int \frac{\sqrt{\mathrm{sech}(2\ln(cx))}}{x^2}\,dx$$

Optimal(type 3, 34 leaves, 5 steps):

$$-\frac{c^2\,x\,\mathrm{arccsch}(c^2x^2)\sqrt{1+\frac{1}{c^4x^4}}\,\sqrt{\mathrm{sech}(2\ln(cx))}}{2}$$

Result(type 8, 15 leaves):

$$\int \frac{\sqrt{\mathrm{sech}(2\ln(cx))}}{x^2}\,dx$$

Problem 49: Unable to integrate problem.

$$\int \mathrm{sech}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3 dx$$

Optimal(type 1, 25 leaves, 4 steps):

$$\frac{2c^2}{e^{3a}\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

Result(type 8, 15 leaves):

$$\int \mathrm{sech}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3 dx$$

Problem 50: Unable to integrate problem.

$$\int \mathrm{sech}\left(a-\frac{\ln(cx^n)}{n(-2+p)}\right)^p dx$$

Optimal(type 3, 66 leaves, 3 steps):

$$\frac{(2-p)\,x\left(1+\frac{1}{e^{2a}\left(c x^{n}\right)^{\frac{2}{n(2-p)}}}\right)\operatorname{sech}\left(a+\frac{\ln(c x^{n})}{n\,(2-p)}\right)^{p}}{2\,(1-p)}$$

Result(type 8, 23 leaves):

$$\int \operatorname{sech}\left(a-\frac{\ln(c x^{n})}{n\,(-2+p)}\right)^{p}\,dx$$

Test results for the 57 problems in "6.5.7 (d hyper)^m (a+b (c sech)^n)^p.txt"

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(d x+c)^{2}}{\left(a+b\operatorname{sech}(d x+c)^{2}\right)^{2}}\,dx$$

Optimal(type 3, 117 leaves, 6 steps):

$$-\frac{(4b+a)\,x}{2a^{3}}+\frac{(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\,\tanh(d x+c)}{\sqrt{a+b}}\right)\sqrt{b}}{2a^{3}d\sqrt{a+b}}+\frac{\cosh(d x+c)\sinh(d x+c)}{2a d\left(a+b-b\tanh(d x+c)^{2}\right)}+\frac{b\tanh(d x+c)}{a^{2}d\left(a+b-b\tanh(d x+c)^{2}\right)}$$

Result(type 3, 536 leaves):

$$-\frac{1}{2 d a^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{2 d a^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}-\frac{2\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)b}{d a^{3}}-\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{2 d a^{2}}$$
$$+\frac{1}{2 d a^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2 d a^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}+\frac{2\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)b}{d a^{3}}+\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2 d a^{2}}$$
$$+\frac{b\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d a^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}b+a+b\right)}$$
$$+\frac{b\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{d a^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}b+a+b\right)}$$
$$+\frac{3\sqrt{b}\,\ln\left(\sqrt{a+b}\,\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2\sqrt{b}\,\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{4 d a^{2}\sqrt{a+b}}$$

$$- \frac{3 \sqrt{b} \ln\left(\sqrt{a+b} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} - 2 \sqrt{b} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right) + \sqrt{a+b} \right)}{4 d a^{2} \sqrt{a+b}}$$

$$+ \frac{b^{3/2} \ln\left(\sqrt{a+b} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} + 2 \sqrt{b} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right) + \sqrt{a+b} \right)}{d a^{3} \sqrt{a+b}}$$

$$- \frac{b^{3/2} \ln\left(\sqrt{a+b} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} - 2 \sqrt{b} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right) + \sqrt{a+b} \right)}{d a^{3} \sqrt{a+b}}$$

```
Problem 13: Result more than twice size of optimal antiderivative.
```

$$\int \frac{\operatorname{csch}(d x + c)^{3}}{\left(a + b \operatorname{sech}(d x + c)^{2} \right)^{2}} \, dx$$

```
Optimal(type 3, 131 leaves, 6 steps):
```

$$\frac{(a - 3 b) \operatorname{arctanh}(\cosh(d x + c))}{2 (a+b)^{3} d} - \frac{(a-b) \cosh(d x + c)}{2 (a+b)^{2} d \left(b + a \cosh(d x + c)^{2} \right)} - \frac{\coth(d x + c) \operatorname{csch}(d x + c)}{2 (a+b) d \left(b + a \cosh(d x + c)^{2} \right)}$$

$$- \frac{(3 a - b) \arctan\left(\frac{\cosh(d x + c) \sqrt{a}}{\sqrt{b}} \right) \sqrt{b}}{2 (a+b)^{3} d \sqrt{a}}$$

```
Result(type 3, 495 leaves):
```

$$\frac{\tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2}}{8 d \left(a^{2} + 2 a b + b^{2} \right)} - \frac{1}{8 d (a+b)^{2} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2}} - \frac{\ln\left(\tanh\left(\frac{d x}{2} + \frac{c}{2} \right) \right) a}{2 d (a+b)^{3}} + \frac{3 \ln\left(\tanh\left(\frac{d x}{2} + \frac{c}{2} \right) \right) b}{2 d (a+b)^{3}}$$

$$+ \frac{b \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} a}{d (a+b)^{3} \left(\tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{4} a + \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{4} b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} b + a + b \right)}$$

$$- \frac{b^{2} \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2}}{d (a+b)^{3} \left(\tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{4} a + \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{4} b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} b + a + b \right)}$$

$$+ \frac{b a}{d (a+b)^{3} \left(\tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{4} a + \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{4} b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2} \right)^{2} b + a + b \right)}$$

$$+\frac{b^2}{d\,(a+b)^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)}$$

$$-\frac{3\,b\arctan\left(\frac{2\,(a+b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+2\,a-2\,b}{4\sqrt{a\,b}}\right)a}{2\,d\,(a+b)^3\sqrt{a\,b}}+\frac{b^2\arctan\left(\frac{2\,(a+b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+2\,a-2\,b}{4\sqrt{a\,b}}\right)}{2\,d\,(a+b)^3\sqrt{a\,b}}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int\frac{\sinh(d x+c)^4}{\left(a+b\,\mathrm{sech}(d x+c)^2\right)^3}\,dx$$

Optimal(type 3, 222 leaves, 8 steps):

$$\frac{3\,(a^2+12\,a\,b+16\,b^2)\,x}{8\,a^5}-\frac{3\,(5\,a^2+20\,a\,b+16\,b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\,\tanh(d x+c)}{\sqrt{a+b}}\right)\sqrt{b}}{8\,a^5\,d\sqrt{a+b}}-\frac{(5\,a+8\,b)\cosh(d x+c)\sinh(d x+c)}{8\,a^2\,d\,\left(a+b-b\tanh(d x+c)^2\right)^2}$$

$$+\frac{\cosh(d x+c)^3\sinh(d x+c)}{4\,a\,d\,\left(a+b-b\tanh(d x+c)^2\right)^2}-\frac{b\,(7\,a+12\,b)\tanh(d x+c)}{8\,a^3\,d\,\left(a+b-b\tanh(d x+c)^2\right)^2}-\frac{3\,b\,(a+2\,b)\tanh(d x+c)}{2\,a^4\,d\,\left(a+b-b\tanh(d x+c)^2\right)}$$

Result(type 3, 1667 leaves):

$$\frac{3\,b}{2\,d\,a^4\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{3\,b}{2\,d\,a^4\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}+\frac{9\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)b}{2\,d\,a^4}+\frac{6\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)b^2}{d\,a^5}$$

$$-\frac{3\,b}{2\,d\,a^4\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\,b}{2\,d\,a^4\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}-\frac{9\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)b}{2\,d\,a^4}-\frac{6\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)b^2}{d\,a^5}$$

$$+\frac{3\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{8\,d\,a^3}-\frac{3\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{8\,d\,a^3}-\frac{1}{4\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{2\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^3}$$

$$+\frac{1}{8\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{3}{8\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{4\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^3}$$

$$-\frac{1}{8\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3}{8\,d\,a^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}$$

$$- \frac{9 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{4 d a^{2} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$- \frac{21 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{4 d a^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$- \frac{3 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{d a^{4} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$- \frac{27 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{4 d a^{2} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$- \frac{35 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{4 d a^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$+ \frac{3 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{d a^{4} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$- \frac{27 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{4 d a^{2} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$- \frac{35 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{4 d a^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$+ \frac{3 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d a^{4} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$

$$
-\frac{9 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d a^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}
$$

$$
-\frac{21 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d a^{3}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}
$$

$$
-\frac{3 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{d a^{4}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}
$$

$$
+\frac{15 \sqrt{b}\, \ln\left(\sqrt{a+b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-2 \sqrt{b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{16 d a^{3} \sqrt{a+b}}
$$

$$
-\frac{15 \sqrt{b}\, \ln\left(\sqrt{a+b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2 \sqrt{b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{16 d a^{3} \sqrt{a+b}}
$$

$$
-\frac{15 b^{3/2} \ln\left(\sqrt{a+b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2 \sqrt{b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{4 d a^{4} \sqrt{a+b}}
$$

$$
+\frac{15 b^{3/2} \ln\left(\sqrt{a+b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-2 \sqrt{b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{4 d a^{4} \sqrt{a+b}}
$$

$$
-\frac{3 b^{5/2} \ln\left(\sqrt{a+b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2 \sqrt{b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{d a^{5} \sqrt{a+b}}
$$

$$
+\frac{3 b^{5/2} \ln\left(\sqrt{a+b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-2 \sqrt{b}\, \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{d a^{5} \sqrt{a+b}}
$$

Problem 15: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{csch}(d x+c)}{\left(a+b \operatorname{sech}(d x+c)^{2}\right)^{3}}\, dx
$$

Optimal(type 3, 140 leaves, 6 steps):

$$-\frac{\operatorname{arctanh}(\cosh(dx+c))}{(a+b)^3 d}-\frac{b\cosh(dx+c)^3}{4a(a+b)d\left(b+a\cosh(dx+c)^2\right)^2}-\frac{b(7a+3b)\cosh(dx+c)}{8a^2(a+b)^2 d\left(b+a\cosh(dx+c)^2\right)}$$
$$+\frac{\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\cosh(dx+c)\sqrt{a}}{\sqrt{b}}\right)\sqrt{b}}{8a^{5/2}(a+b)^3 d}$$

```
Result(type 3, 1475 leaves):
```

$$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a+b)^3}-\frac{9ba\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$+\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$+\frac{13b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2a}$$
$$+\frac{3b^4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2a^2}$$
$$-\frac{27b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a}{4d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$+\frac{9b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$-\frac{21b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2a}$$

$$
- \frac{9 b^{4} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} a^{2}}
- \frac{27 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}
- \frac{13 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}
+ \frac{23 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} a}
+ \frac{9 b^{4} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} a^{2}}
- \frac{9 b a}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}
- \frac{21 b^{2}}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}
- \frac{15 b^{3}}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} a}
- \frac{3 b^{4}}{4 d \left(a+b\right)^{3} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} a^{2}}
+ \frac{15 b \arctan\left(\frac{2 \left(a+b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2 a-2 b}{4 \sqrt{a b}}\right)}{8 d \left(a+b\right)^{3} \sqrt{a b}}
+ \frac{5 b^{2} \arctan\left(\frac{2 \left(a+b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2 a-2 b}{4 \sqrt{a b}}\right)}{4 d \left(a+b\right)^{3} a \sqrt{a b}}
$$

$$+\frac{3 b^{3} \arctan\left(\frac{2 (a+b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2 a-2 b}{4 \sqrt{a b}}\right)}{8 d (a+b)^{3} a^{2} \sqrt{a b}}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}(d x+c)^{3}}{\left(a+b \operatorname{sech}(d x+c)^{2}\right)^{3}}\, dx$$

Optimal(type 3, 195 leaves, 7 steps):

$$\frac{(a-5 b) \operatorname{arctanh}(\cosh(d x+c))}{2 (a+b)^{4} d}+\frac{(2 a-b) b \cosh(d x+c)}{4 a (a+b)^{2} d \left(b+a \cosh(d x+c)^{2}\right)^{2}}-\frac{\left(4 a^{2}-9 a b-b^{2}\right) \cosh(d x+c)}{8 a (a+b)^{3} d \left(b+a \cosh(d x+c)^{2}\right)}$$
$$-\frac{\cosh(d x+c) \coth(d x+c)^{2}}{2 (a+b) d \left(b+a \cosh(d x+c)^{2}\right)^{2}}-\frac{\left(15 a^{2}-10 a b-b^{2}\right) \arctan\left(\frac{\cosh(d x+c) \sqrt{a}}{\sqrt{b}}\right) \sqrt{b}}{8 a^{3/2} (a+b)^{4} d}$$

Result(type 3, 1554 leaves):

$$\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{8 d \left(a^{3}+3 b a^{2}+3 b^{2} a+b^{3}\right)}-\frac{1}{8 d (a+b)^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}-\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right) a}{2 d (a+b)^{4}}+\frac{5 \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right) b}{2 d (a+b)^{4}}$$
$$+\frac{9 b a^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{4 d (a+b)^{4} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$
$$-\frac{5 b^{2} a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{4 d (a+b)^{4} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$
$$-\frac{13 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{4 d (a+b)^{4} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2}}$$
$$+\frac{b^{4} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{4 d (a+b)^{4} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} a}$$

$$+\frac{27 b a^2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$-\frac{21 b^2 a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$+\frac{29 b^3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$-\frac{3 b^4 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 a}$$

$$+\frac{27 b a^2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$+\frac{b^2 a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$-\frac{23 b^3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$+\frac{3 b^4 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 a}$$

$$+\frac{9 b a^2}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$+\frac{17 b^2 a}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$+\frac{7 b^3}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$-\frac{b^4}{4 d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 a}$$

$$-\frac{15 b a \arctan\left(\frac{2 (a+b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{a b}}\right)}{8 d (a+b)^4\sqrt{a b}}+\frac{5 b^2\arctan\left(\frac{2 (a+b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{a b}}\right)}{4 d (a+b)^4\sqrt{a b}}$$

$$+\frac{b^3\arctan\left(\frac{2 (a+b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{a b}}\right)}{8 d (a+b)^4 a\sqrt{a b}}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(d x+c)^4}{a+b\operatorname{sech}(d x+c)^2}\, dx$$

Optimal(type 3, 44 leaves, 3 steps):

$$-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(d x+c)}{\sqrt{a+b}}\right)}{b^{3/2} d\sqrt{a+b}}+\frac{\tanh(d x+c)}{d b}$$

Result(type 3, 137 leaves):

$$\frac{2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{d b\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+1\right)}-\frac{a\ln\left(\sqrt{a+b}\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+2\sqrt{b}\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{2 d b^{3/2}\sqrt{a+b}}$$

$$+\frac{a\ln\left(\sqrt{a+b}\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2-2\sqrt{b}\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{2 d b^{3/2}\sqrt{a+b}}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(dx+c)^5}{a+b\operatorname{sech}(dx+c)^2}\,dx$$

Optimal(type 3, 74 leaves, 5 steps):

$$-\frac{(2a-b)\arctan(\sinh(dx+c))}{2b^2 d}+\frac{a^{3/2}\arctan\left(\frac{\sinh(dx+c)\sqrt{a}}{\sqrt{a+b}}\right)}{b^2 d\sqrt{a+b}}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2db}$$

Result(type 3, 188 leaves):

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}-\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{db^2}$$
$$+\frac{a^{3/2}\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a+b}+2\sqrt{b}}{2\sqrt{a}}\right)}{db^2\sqrt{a+b}}+\frac{a^{3/2}\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a+b}-2\sqrt{b}}{2\sqrt{a}}\right)}{db^2\sqrt{a+b}}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(dx+c)^6}{\left(a+b\operatorname{sech}(dx+c)^2\right)^2}\,dx$$

Optimal(type 3, 89 leaves, 5 steps):

$$-\frac{a(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(dx+c)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d}+\frac{\tanh(dx+c)}{b^2 d}+\frac{a^2\tanh(dx+c)}{2b^2(a+b)d\left(a+b-b\tanh(dx+c)^2\right)}$$

Result(type 3, 1097 leaves):

$$\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{db^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}+\frac{a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{db^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)(a+b)}$$
$$+\frac{a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{db^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)(a+b)}$$
$$-\frac{3a^2\sqrt{ab+b^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+2\sqrt{(a+b)b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a+b\right)}{4db^3(a+b)^2}$$

$$+\frac{3 a^2 \arctan\left(\frac{2 (a+b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right) \sqrt{a b+b^2} \sqrt{(a+b) b}}{2 d b^3 (a+b)^2 \sqrt{a^2+a b}}-\frac{3 a^2 \arctan\left(\frac{2 (a+b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right)}{2 d b^2 (a+b) \sqrt{a^2+a b}}$$

$$-\frac{3 a^2 \sqrt{a b+b^2} \ln\left(-\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+2 \sqrt{(a+b) b} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-a-b\right)}{4 d b^3 (a+b) (-a-b)}$$

$$+\frac{3 a^2 \arctan\left(\frac{2 (-a-b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right)}{2 d b^2 (a+b) \sqrt{a^2+a b}}+\frac{3 a^2 \arctan\left(\frac{2 (-a-b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right) \sqrt{a b+b^2} \sqrt{(a+b) b}}{2 d b^3 (a+b) \sqrt{a^2+a b} (-a-b)}$$

$$-\frac{a \sqrt{a b+b^2} \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+2 \sqrt{(a+b) b} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+a+b\right)}{d b^2 (a+b)^2}$$

$$+\frac{2 a \arctan\left(\frac{2 (a+b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right) \sqrt{a b+b^2} \sqrt{(a+b) b}}{d b^2 (a+b)^2 \sqrt{a^2+a b}}-\frac{2 a \arctan\left(\frac{2 (a+b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right)}{d b (a+b) \sqrt{a^2+a b}}$$

$$-\frac{a \sqrt{a b+b^2} \ln\left(-\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+2 \sqrt{(a+b) b} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-a-b\right)}{d b^2 (a+b) (-a-b)}$$

$$+\frac{2 a \arctan\left(\frac{2 (-a-b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right)}{d b (a+b) \sqrt{a^2+a b}}+\frac{2 a \arctan\left(\frac{2 (-a-b) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b) b}}{2 \sqrt{a^2+a b}}\right) \sqrt{a b+b^2} \sqrt{(a+b) b}}{d b^2 (a+b) \sqrt{a^2+a b} (-a-b)}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(d x+c)^7}{\left(a+b \operatorname{sech}(d x+c)^2\right)^2}\, dx$$

Optimal(type 3, 137 leaves, 6 steps):

$$-\frac{(4 a-b) \arctan(\sinh(d x+c))}{2 b^3 d}+\frac{a^{3/2} (4 a+5 b) \arctan\left(\frac{\sinh(d x+c) \sqrt{a}}{\sqrt{a+b}}\right)}{2 b^3 (a+b)^{3/2} d}+\frac{a (2 a+b) \sinh(d x+c)}{2 b^2 (a+b) d \left(a+b+a \sinh(d x+c)^2\right)}$$

$$+\frac{\operatorname{sech}(d x+c) \tanh(d x+c)}{2 b d \left(a+b+a \sinh(d x+c)^2\right)}$$

Result(type 3, 539 leaves):

$$-\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d b^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+1\right)^{2}}+\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{d b^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+1\right)^{2}}+\frac{\arctan\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d b^{2}}-\frac{4\arctan\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right) a}{d b^{3}}$$
$$-\frac{a^{2}\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d b^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)(a+b)}$$
$$+\frac{a^{2}\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{d b^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)(a+b)}$$
$$+\frac{4 a^{3}\arctan\left(\frac{2(a+b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2\sqrt{(a+b) b}}{2\sqrt{a^{2}+a b}}\right)}{d b^{3}(2 a+2 b)\sqrt{a^{2}+a b}}-\frac{4 a^{3}\arctan\left(\frac{2(-a-b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2\sqrt{(a+b) b}}{2\sqrt{a^{2}+a b}}\right)}{d b^{3}(2 a+2 b)\sqrt{a^{2}+a b}}$$
$$+\frac{5 a^{2}\arctan\left(\frac{2(a+b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2\sqrt{(a+b) b}}{2\sqrt{a^{2}+a b}}\right)}{d b^{2}(2 a+2 b)\sqrt{a^{2}+a b}}-\frac{5 a^{2}\arctan\left(\frac{2(-a-b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2\sqrt{(a+b) b}}{2\sqrt{a^{2}+a b}}\right)}{d b^{2}(2 a+2 b)\sqrt{a^{2}+a b}}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(d x+c)^{6}}{\left(a+b\operatorname{sech}(d x+c)^{2}\right)^{3}}\, dx$$

Optimal(type 3, 130 leaves, 4 steps):

$$\frac{\left(3 a^{2}+8 a b+8 b^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(d x+c)}{\sqrt{a+b}}\right)}{8 b^{5/2}(a+b)^{5/2} d}-\frac{a\operatorname{sech}(d x+c)^{2}\tanh(d x+c)}{4 b(a+b) d\left(a+b-b\tanh(d x+c)^{2}\right)^{2}}-\frac{3 a(a+2 b)\tanh(d x+c)}{8 b^{2}(a+b)^{2} d\left(a+b-b\tanh(d x+c)^{2}\right)}$$

Result(type ?, 3891 leaves): Display of huge result suppressed!

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(d x+c)^{7}}{\left(a+b\operatorname{sech}(d x+c)^{2}\right)^{3}}\, dx$$

Optimal(type 3, 139 leaves, 6 steps):

$$\frac{\arctan(\sinh(dx+c))}{b^3 d} - \frac{a\sinh(dx+c)}{4b(a+b)d\left(a+b+a\sinh(dx+c)^2\right)^2} - \frac{a(4a+7b)\sinh(dx+c)}{8b^2(a+b)^2 d\left(a+b+a\sinh(dx+c)^2\right)}$$
$$-\frac{\left(8a^2+20ab+15b^2\right)\arctan\left(\frac{\sinh(dx+c)\sqrt{a}}{\sqrt{a+b}}\right)\sqrt{a}}{8b^3(a+b)^{5/2}d}$$

```
Result(type 3, 1448 leaves):
```

$$\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d b^3} + \frac{a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 b^2(a+b)}$$
$$+\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 b(a+b)}$$
$$+\frac{a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 (a+b)^2 b^2}$$
$$-\frac{11a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 (a+b)^2 b}$$
$$-\frac{27a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 (a+b)^2}$$
$$-\frac{a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 (a+b)^2 b^2}$$
$$+\frac{11a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 (a+b)^2 b}$$

$$+\frac{27 a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{4 d \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} (a+b)^{2}}$$

$$-\frac{a^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{d \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} b^{2} (a+b)}$$

$$-\frac{9 a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4} b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+a+b\right)^{2} b (a+b)}$$

$$+\frac{a^{3} \arctan\left(\frac{-2 \sqrt{(a+b)^{3}} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b)^{2} b}}{2 \sqrt{a^{3}+2 b a^{2}+b^{2} a}}\right)}{d b^{3} \sqrt{a^{3}+3 b a^{2}+3 b^{2} a+b^{3}} \sqrt{a^{3}+2 b a^{2}+b^{2} a}}-\frac{a^{3} \arctan\left(\frac{2 \sqrt{(a+b)^{3}} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b)^{2} b}}{2 \sqrt{a^{3}+2 b a^{2}+b^{2} a}}\right)}{d b^{3} \sqrt{a^{3}+3 b a^{2}+3 b^{2} a+b^{3}} \sqrt{a^{3}+2 b a^{2}+b^{2} a}}$$

$$+\frac{5 a^{2} \arctan\left(\frac{-2 \sqrt{(a+b)^{3}} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b)^{2} b}}{2 \sqrt{a^{3}+2 b a^{2}+b^{2} a}}\right)}{2 d b^{2} \sqrt{a^{3}+3 b a^{2}+3 b^{2} a+b^{3}} \sqrt{a^{3}+2 b a^{2}+b^{2} a}}-\frac{5 a^{2} \arctan\left(\frac{2 \sqrt{(a+b)^{3}} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b)^{2} b}}{2 \sqrt{a^{3}+2 b a^{2}+b^{2} a}}\right)}{2 d b^{2} \sqrt{a^{3}+3 b a^{2}+3 b^{2} a+b^{3}} \sqrt{a^{3}+2 b a^{2}+b^{2} a}}$$

$$+\frac{15 a \arctan\left(\frac{-2 \sqrt{(a+b)^{3}} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b)^{2} b}}{2 \sqrt{a^{3}+2 b a^{2}+b^{2} a}}\right)}{8 d b \sqrt{a^{3}+3 b a^{2}+3 b^{2} a+b^{3}} \sqrt{a^{3}+2 b a^{2}+b^{2} a}}-\frac{15 a \arctan\left(\frac{2 \sqrt{(a+b)^{3}} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2 \sqrt{(a+b)^{2} b}}{2 \sqrt{a^{3}+2 b a^{2}+b^{2} a}}\right)}{8 d b \sqrt{a^{3}+3 b a^{2}+3 b^{2} a+b^{3}} \sqrt{a^{3}+2 b a^{2}+b^{2} a}}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \left(a+b \operatorname{sech}(d x+c)^{2}\right) \tanh(d x+c)^{4}\, dx$$

Optimal(type 3, 44 leaves, 4 steps):

$$a x-\frac{a \tanh(d x+c)}{d}-\frac{a \tanh(d x+c)^{3}}{3 d}+\frac{b \tanh(d x+c)^{5}}{5 d}$$

Result(type 3, 97 leaves):

$$\frac{1}{d}\left(a\left(d x+c-\tanh(d x+c)-\frac{\tanh(d x+c)^{3}}{3}\right)+b\left(-\frac{\sinh(d x+c)^{3}}{2 \cosh(d x+c)^{5}}-\frac{3 \sinh(d x+c)}{8 \cosh(d x+c)^{5}}\right.\right.$$

$$+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\Bigg)\Bigg)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\operatorname{sech}(dx+c)^2\right)^2\tanh(dx+c)^3\,dx$$

Optimal(type 3, 71 leaves, 4 steps):

$$\frac{a^2\ln(\cosh(dx+c))}{d}+\frac{a(a-2b)\operatorname{sech}(dx+c)^2}{2d}+\frac{(2a-b)b\operatorname{sech}(dx+c)^4}{4d}+\frac{b^2\operatorname{sech}(dx+c)^6}{6d}$$

Result(type 3, 149 leaves):

$$\frac{a^2\ln(\cosh(dx+c))}{d}-\frac{\tanh(dx+c)^2a^2}{2d}-\frac{ab\sinh(dx+c)^2}{2d\cosh(dx+c)^4}+\frac{ab\sinh(dx+c)^2}{2d\cosh(dx+c)^2}-\frac{b^2\sinh(dx+c)^2}{6d\cosh(dx+c)^6}+\frac{b^2\sinh(dx+c)^2}{12d\cosh(dx+c)^4}+\frac{b^2\sinh(dx+c)^2}{12d\cosh(dx+c)^2}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int\coth(dx+c)^4\left(a+b\operatorname{sech}(dx+c)^2\right)^2dx$$

Optimal(type 3, 44 leaves, 4 steps):

$$a^2x-\frac{(a^2-b^2)\coth(dx+c)}{d}-\frac{(a+b)^2\coth(dx+c)^3}{3d}$$

Result(type 3, 95 leaves):

$$\frac{1}{d}\Bigg(a^2\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+2ab\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{2}\right)+b^2\Bigg(\frac{2}{3}$$

$$-\frac{\operatorname{csch}(dx+c)^2}{3}\Bigg)\coth(dx+c)\Bigg)$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int\coth(dx+c)^6\left(a+b\operatorname{sech}(dx+c)^2\right)^2dx$$

Optimal(type 3, 60 leaves, 4 steps):

$$a^2x-\frac{a^2\coth(dx+c)}{d}-\frac{(a^2-b^2)\coth(dx+c)^3}{3d}-\frac{(a+b)^2\coth(dx+c)^5}{5d}$$

Result(type 3, 162 leaves):

$$\frac{1}{d}\left(a^2\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}-\frac{\coth(dx+c)^5}{5}\right)+2ab\left(-\frac{\cosh(dx+c)^3}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}\right.\right.$$
$$\left.+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)}{8}\right)+b^2\left(-\frac{\cosh(dx+c)}{4\sinh(dx+c)^5}\right.$$
$$\left.\left.-\frac{\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)}{4}\right)\right)$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\operatorname{sech}(dx+c)^2\right)^3\tanh(dx+c)^4\,dx$$

Optimal(type 3, 102 leaves, 4 steps):

$$a^3x-\frac{a^3\tanh(dx+c)}{d}-\frac{a^3\tanh(dx+c)^3}{3d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh(dx+c)^5}{5d}-\frac{b^2\left(3a+2b\right)\tanh(dx+c)^7}{7d}+\frac{b^3\tanh(dx+c)^9}{9d}$$

Result(type 3, 273 leaves):

$$\frac{1}{d}\left(a^3\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^3}{3}\right)+3ba^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}\right.\right.$$
$$\left.+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)+3b^2a\left(-\frac{\sinh(dx+c)^3}{4\cosh(dx+c)^7}-\frac{\sinh(dx+c)}{8\cosh(dx+c)^7}\right.$$
$$\left.+\frac{\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(dx+c)^4}{35}+\frac{8\operatorname{sech}(dx+c)^2}{35}\right)\tanh(dx+c)}{8}\right)+b^3\left(-\frac{\sinh(dx+c)^3}{6\cosh(dx+c)^9}-\frac{\sinh(dx+c)}{16\cosh(dx+c)^9}\right.$$
$$\left.\left.+\frac{\left(\frac{128}{315}+\frac{\operatorname{sech}(dx+c)^8}{9}+\frac{8\operatorname{sech}(dx+c)^6}{63}+\frac{16\operatorname{sech}(dx+c)^4}{105}+\frac{64\operatorname{sech}(dx+c)^2}{315}\right)\tanh(dx+c)}{16}\right)\right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\operatorname{sech}(dx+c)^2\right)^3\tanh(dx+c)^2\,dx$$

Optimal(type 3, 86 leaves, 4 steps):

$$a^3x-\frac{a^3\tanh(dx+c)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh(dx+c)^3}{3d}-\frac{b^2\left(3a+2b\right)\tanh(dx+c)^5}{5d}+\frac{b^3\tanh(dx+c)^7}{7d}$$

Result(type 3, 179 leaves):

$$\frac{1}{d}\left(a^3\left(dx+c-\tanh(dx+c)\right)+3\,b\,a^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3\,b^2\,a\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+b^3\left(-\frac{\sinh(dx+c)}{6\cosh(dx+c)^7}+\frac{\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(dx+c)^4}{35}+\frac{8\operatorname{sech}(dx+c)^2}{35}\right)\tanh(dx+c)}{6}\right)\right)$$

```
Problem 40: Result more than twice size of optimal antiderivative.
```

$$\int \coth(dx+c)^5\left(a+b\operatorname{sech}(dx+c)^2\right)^3\,dx$$

```
Optimal(type 3, 77 leaves, 4 steps):
```

$$-\frac{(2a-b)\,(a+b)^2\operatorname{csch}(dx+c)^2}{2d}-\frac{(a+b)^3\operatorname{csch}(dx+c)^4}{4d}-\frac{b^3\ln(\cosh(dx+c))}{d}+\frac{\left(a^3+b^3\right)\ln(\sinh(dx+c))}{d}$$

```
Result(type 3, 193 leaves):
```

$$\frac{a^3\ln(\sinh(dx+c))}{d}-\frac{a^3\coth(dx+c)^2}{2d}-\frac{a^3\coth(dx+c)^4}{4d}-\frac{3\,b\,a^2\cosh(dx+c)^2}{4d\sinh(dx+c)^4}-\frac{3\,b\,a^2\cosh(dx+c)^2}{4d\sinh(dx+c)^2}-\frac{3\,b^2\,a\cosh(dx+c)^2}{4d\sinh(dx+c)^4}+\frac{3\,b^2\,a\cosh(dx+c)^2}{4d\sinh(dx+c)^2}-\frac{b^3}{4d\sinh(dx+c)^4}+\frac{b^3}{2d\sinh(dx+c)^2}+\frac{b^3\ln(\tanh(dx+c))}{d}$$

```
Problem 42: Result more than twice size of optimal antiderivative.
```

$$\int \frac{\coth(dx+c)^4}{a+b\operatorname{sech}(dx+c)^2}\,dx$$

```
Optimal(type 3, 77 leaves, 7 steps):
```

$$\frac{x}{a}-\frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\,\tanh(dx+c)}{\sqrt{a+b}}\right)}{a\,(a+b)^{5/2}\,d}-\frac{(a+2b)\coth(dx+c)}{(a+b)^2\,d}-\frac{\coth(dx+c)^3}{3\,(a+b)\,d}$$

```
Result(type 3, 297 leaves):
```

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{24\,d\,(a+b)^2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{24\,d\,(a+b)^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{8\,d\,(a+b)^2}-\frac{9\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\,d\,(a+b)^2}-\frac{1}{24\,d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3(a+b)}$$

$$-\frac{5a}{8d(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{9b}{8d(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$$
$$-\frac{b^{5/2}\ln\left(\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\sqrt{b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{2da(a+b)^{5/2}}$$
$$+\frac{b^{5/2}\ln\left(\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sqrt{b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{2da(a+b)^{5/2}}$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int\frac{\tanh(dx+c)^5}{(a+b\,\mathrm{sech}(dx+c)^2)^2}\,dx$$

Optimal(type 3, 72 leaves, 4 steps):

$$\frac{(a+b)^2}{2a^2bd\left(b+a\cosh(dx+c)^2\right)}+\frac{\ln(\cosh(dx+c))}{b^2d}+\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\ln\left(b+a\cosh(dx+c)^2\right)}{2d}$$

Result(type 3, 350 leaves):

$$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}{db^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da^2}$$
$$-\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}$$
$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}{2db^2}$$
$$-\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}$$
$$+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}{2da^2}$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(dx+c)^2}{\left(a+b\,\mathrm{sech}(dx+c)^2\right)^2}\,dx$$

Optimal(type 3, 107 leaves, 7 steps):

$$\frac{x}{a^2}-\frac{b^{3/2}\,(5a+2b)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\,\tanh(dx+c)}{\sqrt{a+b}}\right)}{2a^2\,(a+b)^{5/2}\,d}-\frac{(2a-b)\,\coth(dx+c)}{2a\,(a+b)^2\,d}-\frac{b\coth(dx+c)}{2a\,(a+b)\,d\,\left(a+b-b\tanh(dx+c)^2\right)}$$

Result(type 3, 474 leaves):

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a^2+2ab+b^2\right)}-\frac{1}{2d\,(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}$$

$$-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d\,(a+b)^2\,a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}$$

$$-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,(a+b)^2\,a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}$$

$$-\frac{5b^{3/2}\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{4d\,(a+b)^{5/2}\,a}$$

$$+\frac{5b^{3/2}\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{4d\,(a+b)^{5/2}\,a}$$

$$-\frac{b^{5/2}\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{2d\,(a+b)^{5/2}\,a^2}$$

$$+\frac{b^{5/2}\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{2d\,(a+b)^{5/2}\,a^2}$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(dx+c)^3}{\left(a+b\,\mathrm{sech}(dx+c)^2\right)^2}\,dx$$

Optimal(type 3, 104 leaves, 4 steps):

$$\frac{b^3}{2a^2(a+b)^2 d\left(b+a\cosh(dx+c)^2\right)}-\frac{\operatorname{csch}(dx+c)^2}{2(a+b)^2 d}+\frac{b^2(3a+b)\ln\left(b+a\cosh(dx+c)^2\right)}{2a^2(a+b)^3 d}+\frac{(a+3b)\ln(\sinh(dx+c))}{(a+b)^3 d}$$

Result(type 3, 366 leaves):

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8d\left(a^2+2ab+b^2\right)}-\frac{1}{8d(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d(a+b)^3}+\frac{3\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d(a+b)^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da^2}$$
$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da^2}-\frac{2b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}$$
$$+\frac{3b^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}{2da(a+b)^3}$$
$$+\frac{b^3\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}{2da^2(a+b)^3}$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int\frac{\coth(dx+c)}{\left(a+b\operatorname{sech}(dx+c)^2\right)^3}\,dx$$

Optimal(type 3, 124 leaves, 4 steps):

$$-\frac{b^3}{4a^3(a+b)d\left(b+a\cosh(dx+c)^2\right)^2}+\frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(b+a\cosh(dx+c)^2\right)}+\frac{b\left(3a^2+3ab+b^2\right)\ln\left(b+a\cosh(dx+c)^2\right)}{2a^3(a+b)^3d}+\frac{\ln(\sinh(dx+c))}{(a+b)^3d}$$

Result(type 3, 1045 leaves):

$$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a+b)^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da^3}$$
$$-\frac{6b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$-\frac{8b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2a}$$

$$- \frac{2 b^4 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^6}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)^2 a^2}$$

$$- \frac{12 b^2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)^2}$$

$$+ \frac{4 b^3 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)^2 a}$$

$$+ \frac{4 b^4 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)^2 a^2}$$

$$- \frac{6 b^2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)^2}$$

$$- \frac{8 b^3 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)^2 a}$$

$$- \frac{2 b^4 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)^2 a^2}$$

$$+ \frac{3 b \ln\left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)}{2 d a (a+b)^3}$$

$$+ \frac{3 b^2 \ln\left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)}{2 d a^2 (a+b)^3}$$

$$+ \frac{b^3 \ln\left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 a + \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^4 b + 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 a - 2 \tanh\left(\frac{d x}{2} + \frac{c}{2}\right)^2 b + a + b\right)}{2 d a^3 (a+b)^3}$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(dx+c)^3}{\left(a+b\,\text{sech}(dx+c)^2\right)^3}\, dx$$

Optimal(type 3, 146 leaves, 4 steps):

$$-\frac{b^4}{4a^3(a+b)^2 d\left(b+a\cosh(dx+c)^2\right)^2}+\frac{b^3(2a+b)}{a^3(a+b)^3 d\left(b+a\cosh(dx+c)^2\right)}-\frac{\text{csch}(dx+c)^2}{2(a+b)^3 d}+\frac{b^2\left(6a^2+4ab+b^2\right)\ln\left(b+a\cosh(dx+c)^2\right)}{2a^3(a+b)^4 d}+\frac{(4b+a)\ln(\sinh(dx+c))}{(a+b)^4 d}$$

Result(type 3, 1127 leaves):

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8d\left(a^3+3ba^2+3b^2a+b^3\right)}-\frac{1}{8d(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d(a+b)^4}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d(a+b)^4}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da^3}$$
$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da^3}-\frac{8b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$-\frac{10b^4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2a}$$
$$-\frac{2b^5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da^2(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$-\frac{16b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$
$$+\frac{8b^4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2a}$$
$$+\frac{4b^5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da^2(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$$

$$-\frac{8 b^3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$-\frac{10 b^4 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{d (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2 a}$$

$$-\frac{2 b^5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{d a^2 (a+b)^4 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)^2}$$

$$+\frac{3 b^2 \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)}{d a (a+b)^4}$$

$$+\frac{2 b^3 \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)}{d a^2 (a+b)^4}$$

$$+\frac{b^4 \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4 b+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 b+a+b\right)}{2 d a^3 (a+b)^4}$$

```
Problem 49: Unable to integrate problem.
```

$$\int \left(a+b\,\mathrm{sech}(x)^2\right)^{3/2} \tanh(x)^2\,\mathrm{d}x$$

```
Optimal(type 3, 103 leaves, 9 steps):
```

$$a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)-\frac{(3 a^2-6 a b-b^2) \arctan\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)}{8\sqrt{b}}-\frac{(5 a+b)\sqrt{a+b-b\tanh(x)^2}\,\tanh(x)}{8}$$

$$+\frac{b\sqrt{a+b-b\tanh(x)^2}\,\tanh(x)^3}{4}$$

```
Result(type 8, 17 leaves):
```

$$\int \left(a+b\,\mathrm{sech}(x)^2\right)^{3/2} \tanh(x)^2\,\mathrm{d}x$$

```
Problem 50: Unable to integrate problem.
```

$$\int \left(a+b\,\mathrm{sech}(x)^2\right)^{3/2} dx$$

Optimal(type 3, 70 leaves, 7 steps):

$$a^{3/2}\,\mathrm{arctanh}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)+\frac{(3a+b)\arctan\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)\sqrt{b}}{2}+\frac{b\sqrt{a+b-b\tanh(x)^2}\,\tanh(x)}{2}$$

Result(type 8, 12 leaves):

$$\int \left(a+b\,\mathrm{sech}(x)^2\right)^{3/2} dx$$

Problem 51: Unable to integrate problem.

$$\int \frac{\tanh(x)^2}{\sqrt{a+b\,\mathrm{sech}(x)^2}}\, dx$$

Optimal(type 3, 48 leaves, 7 steps):

$$\frac{\mathrm{arctanh}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)}{\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)}{\sqrt{b}}$$

Result(type 8, 17 leaves):

$$\int \frac{\tanh(x)^2}{\sqrt{a+b\,\mathrm{sech}(x)^2}}\, dx$$

Problem 52: Unable to integrate problem.

$$\int \frac{\coth(x)}{\sqrt{a+b\,\mathrm{sech}(x)^2}}\, dx$$

Optimal(type 3, 44 leaves, 7 steps):

$$\frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(x)^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(x)^2}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Result(type 8, 15 leaves):

$$\int \frac{\coth(x)}{\sqrt{a+b\,\mathrm{sech}(x)^2}}\, dx$$

Problem 53: Unable to integrate problem.

$$\int \frac{\tanh(x)^3}{\left(a+b\,\mathrm{sech}(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 44 leaves, 5 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(x)^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{-a-b}{a\,b\,\sqrt{a+b\,\mathrm{sech}(x)^2}}$$

Result(type 8, 17 leaves):

$$\int \frac{\tanh(x)^3}{\left(a+b\,\mathrm{sech}(x)^2\right)^{3/2}}\,dx$$

Problem 55: Unable to integrate problem.

$$\int \frac{1}{\left(a+b\,\mathrm{sech}(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 49 leaves, 4 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a\,(a+b)\,\sqrt{a+b-b\tanh(x)^2}}$$

Result(type 8, 12 leaves):

$$\int \frac{1}{\left(a+b\,\mathrm{sech}(x)^2\right)^{3/2}}\,dx$$

Problem 56: Unable to integrate problem.

$$\int \frac{1}{\left(a+b\,\mathrm{sech}(x)^2\right)^{5/2}}\,dx$$

Optimal(type 3, 81 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh(x)^2}}\right)}{a^{5/2}}-\frac{b\,(5a+3b)\tanh(x)}{3a^2\,(a+b)^2\sqrt{a+b-b\tanh(x)^2}}-\frac{b\tanh(x)}{3a\,(a+b)\,\left(a+b-b\tanh(x)^2\right)^{3/2}}$$

Result(type 8, 12 leaves):

$$\int \frac{1}{\left(a+b\,\mathrm{sech}(x)^2\right)^{5/2}}\,dx$$

Problem 57: Unable to integrate problem.

$$\int \frac{1}{\left(a+b\operatorname{sech}(dx+c)^2\right)^{7/2}}\, dx$$

Optimal(type 3, 165 leaves, 7 steps):

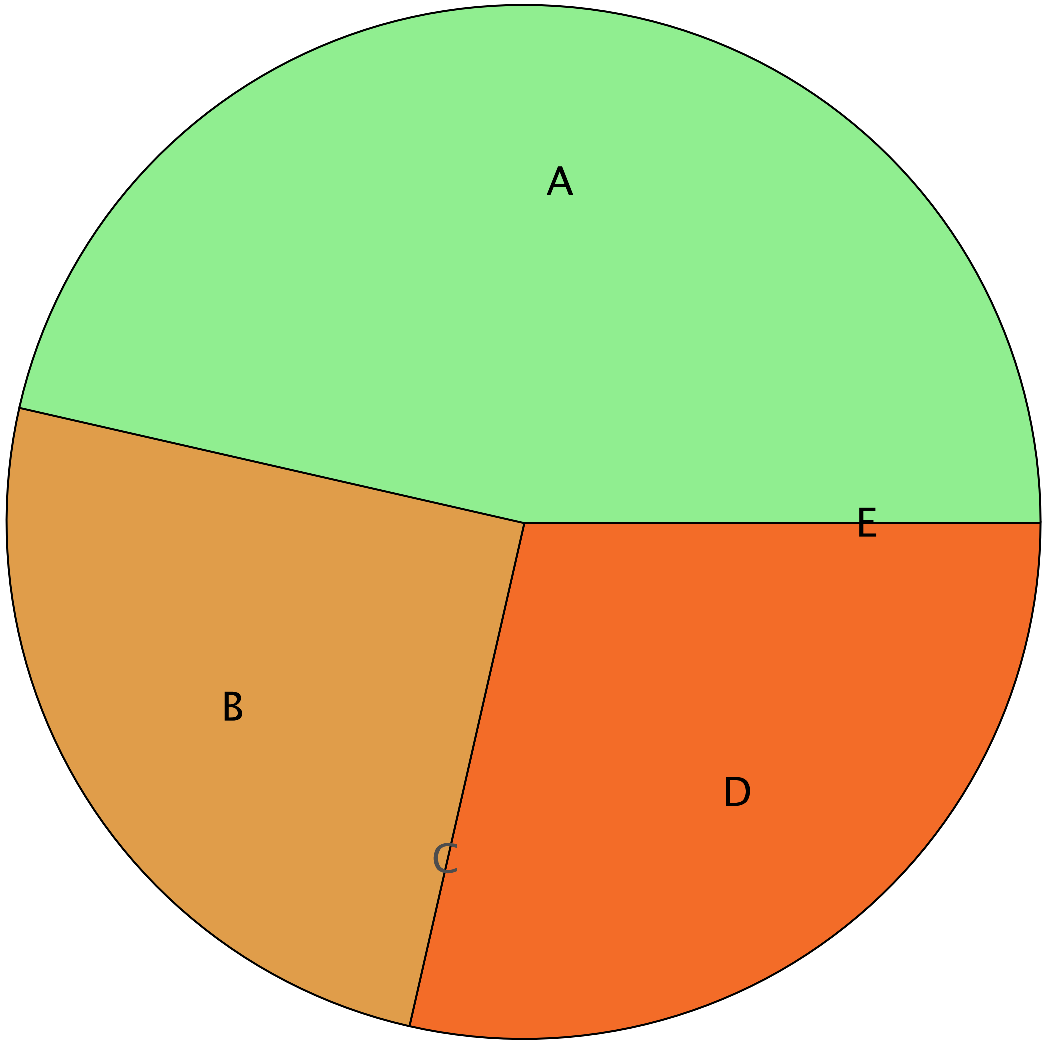
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\,\tanh(dx+c)}{\sqrt{a+b-b\tanh(dx+c)^2}}\right)}{a^{7/2}\,d}-\frac{b\left(33a^2+40ab+15b^2\right)\tanh(dx+c)}{15a^3\,(a+b)^3\,d\sqrt{a+b-b\tanh(dx+c)^2}}-\frac{b\tanh(dx+c)}{5a\,(a+b)\,d\left(a+b-b\tanh(dx+c)^2\right)^{5/2}}$$
$$-\frac{b\,(9a+5b)\tanh(dx+c)}{15a^2\,(a+b)^2\,d\left(a+b-b\tanh(dx+c)^2\right)^{3/2}}$$

Result(type 8, 16 leaves):

$$\int \frac{1}{\left(a+b\operatorname{sech}(dx+c)^2\right)^{7/2}}\, dx$$

Summary of Integration Test Results

140 integration problems



A - 65 optimal antiderivatives
B - 35 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 40 unable to integrate problems
E - 0 integration timeouts